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Search and Enumeration Techniques for Incidence Structures

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Chapter 3

Exhaustive Construction Of Incidence Structures

3.1 INTRODUCTION

Exhaustive search is a technique for constructing or examining all possible states within a given search space. The most well-known general exhaustive search strategy is that of backtracking, in which partial solutions are built up one step at a time in a systematic fashion guaranteeing complete coverage of the search space. In contrast, non-exhaustive search strategies, such as the probabilistic algorithms studied in the previous chapter, traverse the search space more or less at random and thus certain states may never be examined.

The fundamental differences between exhaustive and non-exhaustive strategies are apparent by considering the advantages of each approach: non-exhaustive search is generally faster, whereas exhaustive search is more thorough.

An exhaustive search strategy is usually required for any type of problem which poses questions such as "in how many ways can this be solved?", or "how many designs exist with a particular structure?". In many cases, the only way of answering such questions is to explicitly construct all the solutions. Exhaustive search can also be used to establish the non-existence of particular structures, by exhaustively searching the corresponding search space, without producing any valid designs. Good examples of such applications of exhaustive search are the celebrated work by Lam, Thiel and Swiercz [42] who establish the non-existence of finite projective planes of order 10, the work of McKay and Radziszowski [49] who show that no 4-(12,6,6) designs exist, and even the work presented in Case Study One of Chapter 6 of this thesis, in which it is shown that weakly union free twofold triple systems of order 12 do not exist.

The limited success of the non-exhaustive, probabilistic techniques in the previous chapter, particularly for problems with very large search spaces in which valid solutions are rare or non-existent, motivated the development of an exhaustive search algorithm for general incidence structures. The algorithm was based on the technique of backtracking and was developed in several stages, with extensive analysis and progressive refinements being made at each stage. The final version of the algorithm produced a number of new results in the field of combinatorial design theory, which are covered in their relevant sections of this thesis, and summarised in Chapter 7.

This chapter describes the underlying methods and some of the optimisation techniques of the incidence structure backtracking algorithm. In particular, the algorithm developed in this chapter is able to exhaustively generate all possible incidence structures of a given type, thus answering the question "how many distinct designs are there satisfying a particular set of properties?"

Section 3.2 presents an overview of the technique of backtracking that formalises its methods and outlines its limitations. This is followed by Section 3.3 which introduces incidence structures in general and presents the special type of incidence structures which are used as examples throughout the thesis, profiling their common representations.